PERGAMON

International Journal of Heat and Mass Transfer 45 (2002) 3199–3204

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

An experimental heat transfer study for periodically varying-curvature curved-pipe

Ru Yang *, Fan Pin Chiang

Department of Mechanical and Electro-Mechanical Engineering, National Sun Yat-Sen University, Kaohsiung 804, Taiwan

Received 18 October 2001

Abstract

Experiments were conducted for water flowing through a varying-curvature curved-pipe inside a larger diameter straight pipe to form a double-pipe heat exchanger with water as the working medium. The heat transfer coefficients were obtained using the Wilson plot method. The effects of the Dean, Prandtl, Reynolds number and the curvature ratio on the average heat transfer coefficients and the friction factors are presented. A higher Dean number results in a higher heat transfer rate. It is found that the heat transfer rate may be increased by up to 100%, as compared with a straight pipe, while the friction coefficient increased by less than 40%. Therefore, it is promising to use S-shaped pipes instead of straight pipes for the performance enhancement for a heat exchanger such as a solar collector.

Keywords: Fluid flow; Heat transfer; Varying-curvature; Curved-pipe; Experiment

1. Introduction

Curved-pipe curvature induces a secondary flow across the main stream that may enhance heat transfer rate significantly [1–6]. Therefore, it should receive much attention in the heat transfer enhancement applications. However, literature for the study of periodically varying-curvature curved-pipes (e.g., wavy curved-pipe) is comparatively very little [7–9]. The perturbation models of [7–9] models are restricted to very small amplitude of wavy pipes that limits its application drastically. In addition, the study of [7] only deals with the flow solution while heat transfer is very important in applications. This study is motivated to provide experimental data for curved-pipe applications.

2. Experimental system

Experiments are made for measuring the heat transfer and the pressure drop for water flow inside a wavy curved-pipe with axial function $y = a\sin(\kappa x)$. The geometry of the pipe is illustrated in Fig. 1. The curved-pipe is placed inside a larger circular pipe to form a double-pipe heat exchanger.

Fig. 2 is the schematic of the experimental system. The lower loop in Fig. 2 is the loop for hot water flowing inside the curved-pipe, whose temperature is controlled by the adjustment of a heater, and flow rate is controlled by the adjustment of a valve and a centrifugal pump. The upper loop provides the cooling water flowing through the space between the curved-pipe and the PVC pipe, with inlet temperature controlled by a constant temperature bath and flow rate controlled by a constant mass flow rate pump (Fasco Dizark). However, these two flow loops are interchangeable so that either heating or cooling of the fluid flow inside the curved-pipe can be tested.

The test section of 1 m long, as illustrated in Fig. 3, consists of a curved-pipe made of 9.52 mm OD copper tube and a 56 mm OD PVC pipe insulated externally.

* Corresponding author. Tel.: +886-7-525-4222; fax: +886-7-525-4299.

E-mail address: yangru@mail.nsysu.edu.tw (R. Yang).

0017-9310/02/$ - see front matter
PII: S0017-9310(02)00023-6

Nomenclature

A	heat transfer area (m^2)
a	amplitude of wavy pipe (cm)
a'	dimensionless a, a/r
C_p	constant pressure specific heat (kJ/kg °C)
D	pipe diameter (mm)
De	Dean number, $Re^2\delta$
f	friction factor, $(D_i/L)(2\Delta p/\rho v^2)$
h	heat transfer coefficient (W/m^2 °C)
k	conductivity (W/m °C)
L	length of test section (cm)
$\dot{m}$	mass flow rate (kg/s)
Nu	Nusselt number $= hD/k$
P	pressure (kPa)
Pr	Prandtl number $= C_p\mu/k$
$\dot{Q}$	heat transfer rate (W)
r	pipe radius (cm)
Re	Reynolds number $= \rho vD/\mu$
R_{wall}	pipe wall thermal resistance $(\mathrm{W}\ °\mathrm{C})^{-1}$
T	temperature (°C)
U	overall heat transfer coefficient (W/m^2 °C)
v	velocity (m/s)
x	x-coordinate
y	y-coordinate

Greek symbols

ΔP	pressure difference (N/m^2)
ΔT_{lm}	log mean temperature difference
δ	curvature ratio $= r\bar{\kappa}_c$
κ	wave number of wavy pipe $= 2\pi/L$ (cm^{-1})
κ'	dimensionless $\kappa = r\kappa$
κ_c	curvature of curved-pipe
$\bar{\kappa}_c$	average curvature of wavy curved-pipe
μ	dynamic viscosity (N s/m^2)
ρ	density (kg/m^3)

Subscripts

av	average quantities
c	cold
h	hot
i	inner
o	outer
s	quantities associated with the straight pipe

The pressure drop for water flowing through the curved-pipe is measured with a differential pressure gauge. All temperatures are measured with RTD thermometers.

3. Analysis for data correlation

The pipe curvature at any location is

$$\kappa_c = \kappa^2 a \sin \kappa x/(1+\kappa^2 a^2 \cos^2 \kappa x)^{1.5}. \tag{1}$$

Heat transfer to the cold water in test section is

$$\dot{Q}_c = \dot{m}_c C_{p,c}(T_{c,o} - T_{c,i}), \tag{2}$$

where $\dot{m}_c$ is the flow rate of cold water, $C_{p,c}$ is the specific heat of the cold water. $T_{c,i}$ and $T_{c,o}$ are, respectively, the inlet and the outlet temperatures of the cold water. Similarly, heat transfer from the hot water in test section is

$$\dot{Q}_h = \dot{m}_h C_{p,h}(T_{h,o} - T_{h,i}), \tag{3}$$

where $\dot{m}_h$ is the flow rate of hot water and $C_{p,h}$ is the specific heat of the hot water. $T_{h,i}$ and $T_{h,o}$ are, respectively, the inlet and the outlet temperatures of hot water. The energy balance for the test section should be $\dot{Q}_c \approx$

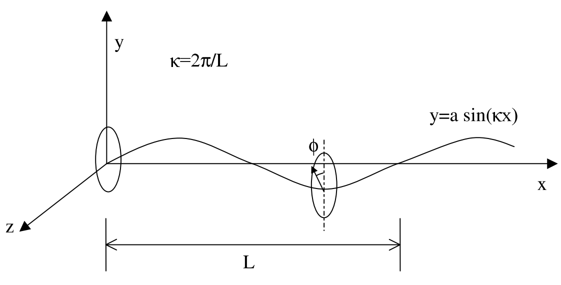

Fig. 1. Geometric shape of wavy curved-pipe.

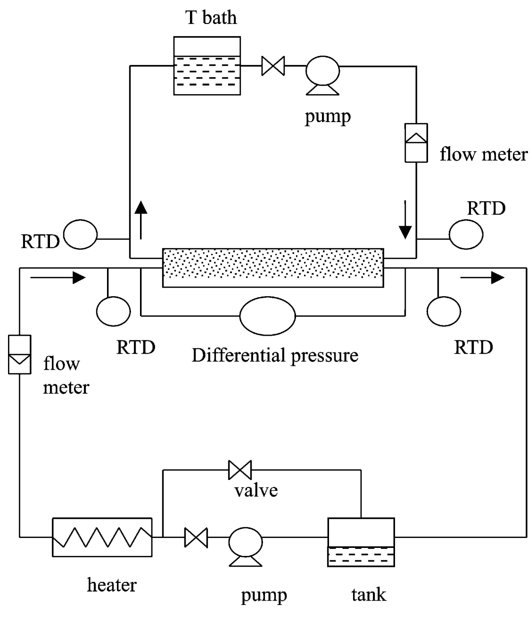

Fig. 2. Schematic of experimental system.

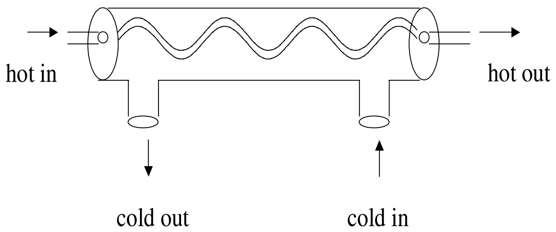

Fig. 3. Schematic of test section.

$-\dot{Q}_h$, or $(\dot{Q}_c + \dot{Q}_h) \to 0$, this provides a check for data accuracy.

Averaged heat transfer rate is defined as

$$\dot{Q}_{av} = \frac{1}{2}\left(|\dot{Q}_h| + |\dot{Q}_c|\right). \tag{4}$$

The corresponding overall heat transfer coefficient U_o is defined as

$$\dot{Q}_{av} \equiv U_o A_o \Delta T_{lm}, \tag{5}$$

where A_o is the total heat transfer area, ΔT_{lm} is the log mean temperature difference (LMTD) defined as

$$\Delta T_{lm}(\mathrm{LMTD}) \equiv \frac{\Delta T_a - \Delta T_b}{\ln(\Delta T_a/\Delta T_b)}, \tag{6}$$

where

$$\Delta T_a \equiv \mathrm{T}_{h,i} - T_{c,o}, \tag{7}$$

$$\Delta T_b \equiv \mathrm{T}_{h,o} - T_{c,i} \tag{8}$$

with subscript "o" and "i" denoting outer and inner surface of the wavy curved-pipe, respectively. Then the overall heat transfer resistance can be expressed by

$$\frac{1}{U_o A_o} = \frac{\Delta T_{lm}}{\dot{Q}_{av}}. \tag{9}$$

This overall resistance consists of three resistances in series: the convective resistance on inner surface, the wavy curved pipe wall conduction resistance, and the convective resistance on outer surface of the wavy curved-pipe. That is

$$\frac{1}{U_o A_o} = \frac{1}{h_o A_o} + R_{wall} + \frac{1}{h_i \mathrm{A_i}}, \tag{10}$$

where h_o and h_i are the convective heat transfer coefficients on outer and inner surface of the wavy curved-pipe, respectively.

Parameters that affects the problem include Reynolds number and the geometric parameters of a and κ that determine the curvature of a curved-pipe.

The inner surface heat transfer coefficient is cast into the form

$$h_i = C_i Re_i^m \frac{k_i}{D_i}, \tag{11}$$

where m and C_i are constants to be determined. The outer surface heat transfer coefficient can be expressed accordingly as

$$h_o = C_o Re_o^{0.8} \frac{k_o}{D_o}. \tag{12}$$

Substituting Eqs. (11) and (12) into Eq. (10), results in

$$\frac{1}{U_o A_o} - R_{wall} = \frac{1}{(C_i Re_i^m (k_i/D_i))A_i} + \frac{1}{(C_o Re_o^{0.8}(k_o/D_o))A_o}. \tag{13}$$

The value of $1/U_o A_o$ can be evaluated by Eq. (9), and R_{wall} can be usually neglected. C_i, m and C_o are the coefficients to be determined. Eq. (13) can be rewritten as

$$\frac{Re_o^{0.8}(k_o/D_o)}{U_o} = \frac{1}{C_i}\frac{Re_o^{0.8}(k_o/D_o)A_o}{Re_i^m (k_i/D_i)A_i} + \frac{1}{C_o}. \tag{14}$$

Reform Eq. (14) into a linear form

$$Y = AX + B, \tag{15}$$

where

$$Y = \frac{Re_o^{0.8}(k_o/D_o)}{U_o},$$

$$X = \frac{Re_o^{0.8}(k_o/D_o)A_o}{Re_i^m (k_i/D_i)A_i},$$

$$A = \frac{1}{C_i}, \quad B = \frac{1}{C_o}.$$

If the value of m is assumed, experimental data can fit Eq. (15) to determine C_i and C_o. This is the well-known Wilson plot method. However, in order to determine m, modified Wilson plot method [10] is employed. Eq. (13) is reformed into

$$C_i Re_i^m = 1\Big/\left[\left(\frac{1}{U_o} - \frac{1}{h_o}\right)\frac{(k_i/D_i)A_i}{A_o}\right] \tag{16}$$

which is then expressed by

$$Y_2 = DX_2 + E, \tag{17}$$

where

$$Y_2 = \ln\left[1\Big/\left\{\left(\frac{1}{U_o} - \frac{1}{h_o}\right)\frac{(k_i/D_i)A_i}{A_o}\right\}\right],$$

$$X_2 = \ln(Re_i),$$

$$D = m, \quad \text{and} \quad E = \ln C_i.$$

Experimental data can fit Eq. (17) to determine m and C_i. The obtained new value of m is then compared with the assumed value of m for Eq. (15), if they are not equal, the value of m for Eq. (15) is reassumed and the procedure is repeated until m, C_i and C_o are consistent. Heat transfer coefficient h_i is obtained thereby.

Dimensionless parameters that affect the flow and heat transfer of the problem including the wavy curved-pipe amplitude, a', wavy number, κ', the flow Reynolds number and the Prandtl number.

4. Results and discussions

Figs. 4 and 5 illustrate the modified Wilson plot of experimental data for various a and κ, respectively, the linearity confirms that the method developed is appropriate.

It is shown in Fig. 6 that heat transfer coefficient increases with increasing κ. The larger the κ, the larger the curvature as indicated in Eq. (1). A larger curvature corresponds to a stronger secondary flow. The heat transfer rate is enhanced therefore. The effect of a on the heat transfer rate, shown in Fig. 7, is similar to that of κ, since increasing a also increases curvature as indicated in Eq. (1). When κ or/and a equal to zero, the wavy curved-pipe curvature would be zero, that means the pipe should be straight. It is clearly shown in Figs. 6 and 7, that curved-pipe has higher heat transfer coefficient than that of a straight pipe. Especially for Re around 3000, the heat transfer rate is increased by 100%.

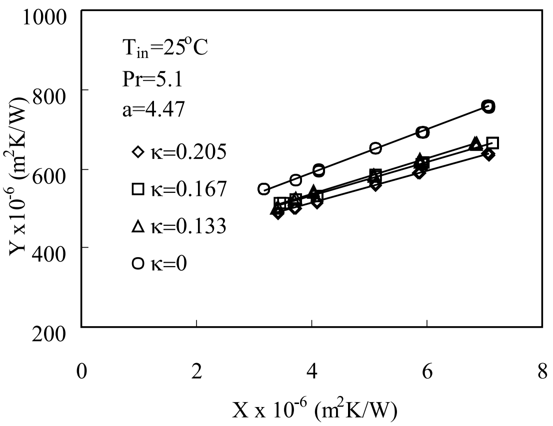

Fig. 4. Wilson plot for various κ.

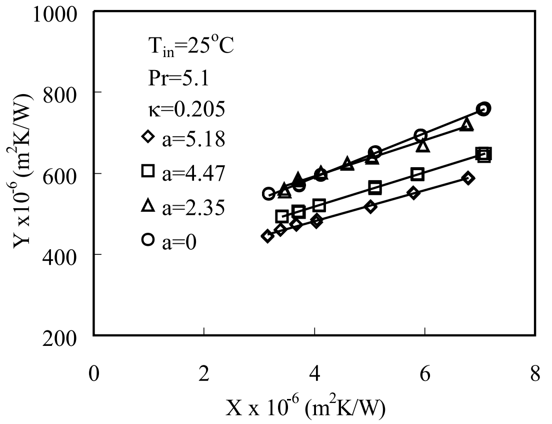

Fig. 5. Wilson plot for various a.

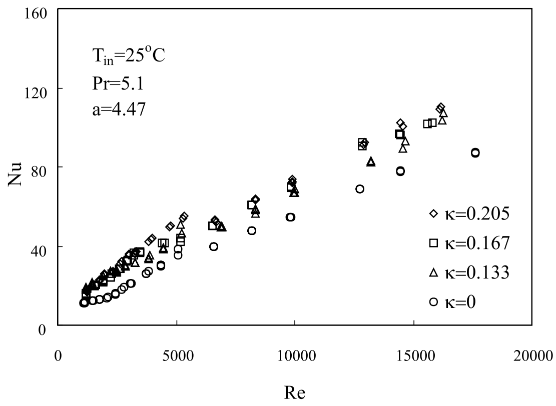

Fig. 6. Reynolds number effect on Nu for various κ for heating process.

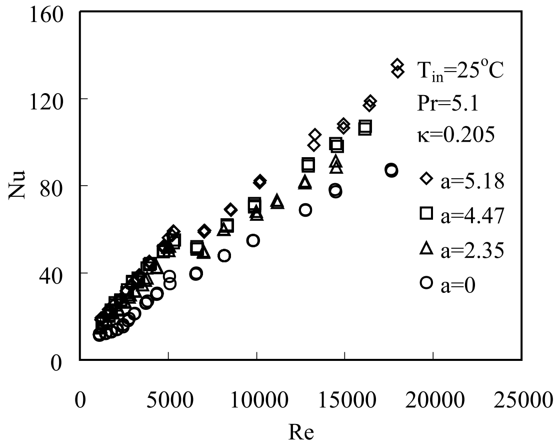

Fig. 7. Reynolds number effect on Nu for various a for heating process.

As expected, pressure drop is higher in a curved-pipe than in a straight pipe. Figs. 8 and 9 show that friction factor, f, increases with increasing curvature parameters of κ and a, respectively. This is owing to the higher strength of the secondary flow corresponding to the higher curvature. As shown, in laminar region, f equals the theoretical value of $64/Re$ for a smooth pipe ($a = 0$ or $\kappa = 0$). In turbulent region, data for smooth pipes are fit well with Blasius' [11] correlation

$$f \cong 0.316Re^{-(1/4)}, \quad Re < 2 \times 10^4, \tag{18}$$

$$f \cong 0.184Re^{-(1/5)}, \quad Re \geqslant 2 \times 10^4. \tag{19}$$

Dimensionless pump power can be expressed by fRe^3 [12]. Figs. 10 and 11 show the effects of κ and a, respectively, on the heat transfer coefficient for various

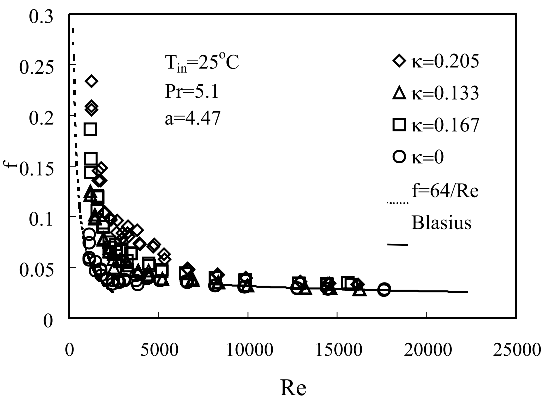

Fig. 8. Reynolds number effect on f for various κ for heating process.

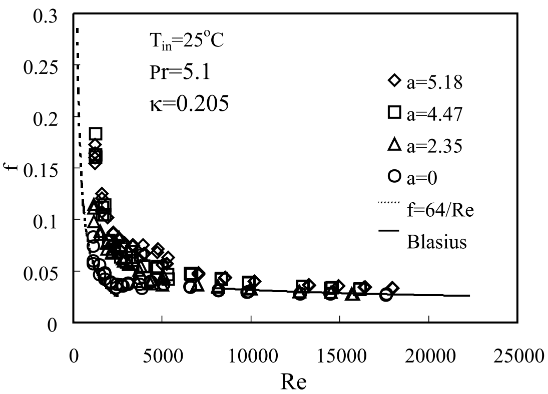

Fig. 9. Reynolds number effect on f for various κ for heating process.

pump power consumption. It can be seen that curved-pipe gives net benefit (up to 100% increase) in heat transfer rate for any fixed pump power consumption.

Data can also be presented in terms of the dimensionless curvature parameters of the Dean number [2], De, and the curvature ratio, δ. Noting that the curvature varies periodically, averaged curvature is employed in calculation, and we find for the cooling case, the curvature ratio effect is not profound, and the heat transfer rate is almost depending on the Dean number only.

The experimental data can be correlated into the following forms.

For turbulent flow ($Re > 2000$):

$$Nu = 2.87De^{0.4}\delta^{-0.203}Pr^{0.386}, \quad R^2 = 0.85,$$

$$f = 1.69De^{-0.159}\delta^{0.488}, \quad R^2 = 0.95$$

for $2.1 \times 10^6 \leqslant De \leqslant 5.5 \times 10^7$, $0.050 < \delta < 0.096$, and $4.0 < Pr < 5.2$.

For laminar flow ($Re < 2000$):

$$Nu = 0.185De^{0.325}\delta^{-0.157}Pr^{0.234}, \quad R^2 = 0.89,$$

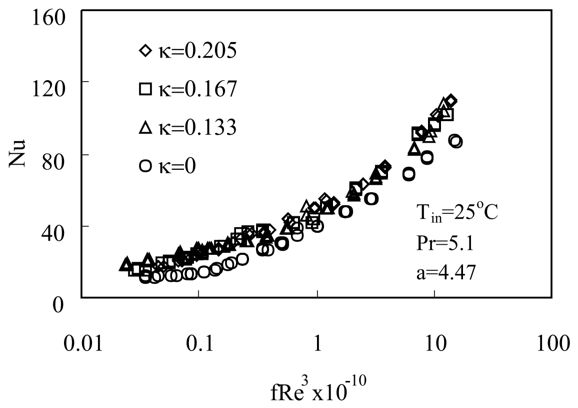

Fig. 10. Pump power effect on Nu for various κ for heating process.

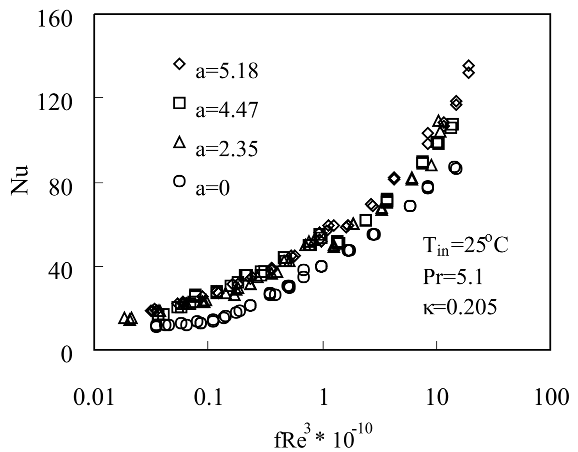

Fig. 11. Pump power effect on Nu for various a for heating process.

$$f = 739De^{-0.507}\delta^{0.988}, \quad R^2 = 0.87$$

for $2.5 \times 10^4 \leqslant De \leqslant 6 \times 10^5$, $0.050 < \delta < 0.096$, and $3.9 < Pr < 4.5$.

5. Conclusions

The present study provides data and correlations for heat transfer and flow friction in wavy curved-pipes. The effect of the Dean, Prandtl, Reynolds number and curvature ratio on the average heat transfer coefficients and the friction factors are presented. A higher Dean number results in a higher heat transfer rate. It is found, in the range of the present tests, that the heat transfer rate may be increased by up to 100%, as compared with a straight pipe, while the friction factor increased by less than 40%. Therefore, it is promising to use S-shaped pipes instead of straight pipes for enhancing heat exchanger or a solar collector performance. Flow ranges of low Reynolds number ($Re < 600$) was not tested due to incompatible experimental setup. Revising the experimental system is under progress and the study will be made in the near future.

Acknowledgements

This study was sponsored by the National Science Council, Taiwan, under the contract NSC88- 2212-E-110-020.

References

[1] J. Eustic, Experiments of streamline motion in curved-pipes, Proc. R. Soc. London Ser. A 85 (1911) 119–131.
[2] W.R. Dean, Note on the motion of fluid in a curved-pipe, Philos. Mag. Ser. 7 (5) (1927) 208–223.
[3] S.A. Berger, L. Talbot, L.S. Tao, Flow in curved-pipe, Annu. Rev. Fluid Mech. 15 (1983) 461–512.
[4] R. Yang, S.F. Chang, Numerical study of fully developed laminar flow and heat transfer in a curved-pipe with arbitrary curvature ratio, Int. J. Heat Fluid Flow (1993).
[5] R. Yang, S.F. Chang, Combined free and forced convection for developed flow in curved-pipe with finite curvature ratio, Int. J. Heat Fluid Flow 15 (6) (1994) 470–476.
[6] R. Yang, S.F. Chang, W. Wu, A numerical study of flow and heat transfer in a curved-pipe with periodically varying curvature, ASME paper 95-WA/HT-9, 1995.
[7] S. Murata, Y. Miyaka, T. Inaba, Laminar flow in a curved-pipe with varying curvature, J. Fluid Mech. 73 (1976) 735–752.
[8] Y. Itmoto, M. Nagata, F. Yamamoto, Steady laminar flow of viscoelastic fluid in a curved-pipe of circular cross-section with varying curvature, J. Non-Newtonian Fluid Mech. 22 (1986) 101–114.
[9] N.P. Gopalan, Laminar flow of a suspension in a curved-pipe with varying curvature, Int. J. Energy 23 (1985) 621–632.
[10] R.K. Shah, Assessment of modified wilson plot techniques for obtaining heat exchanger design data, in: Proceedings of the 9th International Heat Transfer Conference, 5, 1990, pp. 51–56.
[11] H. Blasius, Das ahnlichkeitsgesetz bei reibungsvorgangen in flussigkeiten, Forschungs- arbeiten des Ver. Deutsch. Ing., No. 131, quoted in R.L. Webb, 1987, Handbook of single-phase heat transfer, Chapter 4, S. Kakac, R.K. Shah, W. Aung (Eds.), John Wiley & Sons, New York, 1913.
[12] B. Snyder, K.T. Li, R.A. Wirtz, Heat transfer enhancement in a serpentine channel, Int. J. Heat Mass Transfer 36 (12) (1993) 2965–2976.